① a) $x^2 - y^2 = 2xy$

$2x - 2y\frac{dy}{dx} = 2x \cdot \frac{dy}{dx} + y \cdot 2$

$2x - 2y = 2x\frac{dy}{dx} + 2y\frac{dy}{dx}$

$2x - 2y = \frac{dy}{dx}(2x + 2y)$

$\frac{2x-2y}{2x+2y} = \frac{dy}{dx}$

b) $x^3 + xy + y^3 = 4$

$3x^2 + x\frac{dy}{dx} + y \cdot 1 + 3y^2 \cdot \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-y - 3x^2}{x + 3y^2}$

c) $y^{-1} + x^{-1} = 2$

$-y^{-2} \cdot \frac{dy}{dx} + -x^{-2} = 0$

$-\frac{1}{y^2} \cdot \frac{dy}{dx} - \frac{1}{x^2} = 0$

$-\frac{1}{y^2} \cdot \frac{dy}{dx} = \frac{1}{x^2}$

$\frac{dy}{dx} = \frac{-y^2}{x^2}$

d) $3x^4 = (2xy - 1)^3$

$12x^3 = 3(2xy-1)^2 \cdot (2x\frac{dy}{dx} + y \cdot 2)$

$\frac{12x^3 - 6y(2xy-1)^2}{6x(2xy-1)^2} = \frac{dy}{dx}$

② a) $y^2 = \frac{x^2 - 4}{x^2 + 4}$

$\frac{1}{2y} \cdot 2y\frac{dy}{dx} = \frac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2} \cdot \frac{1}{2y}$

$\frac{dy}{dx} = \frac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2 \cdot 2y}$

$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{8 \cdot 4 - 0}{8^2 \cdot 0}$ undef.

$y - 0 = \frac{32}{0}(x - 2)$

tangent: $x = 2$

~~Normal: y~~

b) $(x+y)^3 = x^3 + y^3$

$3(x+y)^2(1 + \frac{dy}{dx}) = 3x^2 + 3y^2 \cdot \frac{dy}{dx}$

plug in $(-1,1)$ → $3(0)^2 \cdot (1 + \frac{dy}{dx}) = 3(-1)^2 + 3(1)^2 \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = -1$

tangent ~~normal~~ $\boxed{y - 1 = -1(x + 1)}$

$(x+y)^3 = x^3 + y^3$

$3(x+y)^2 \cdot (1 + \frac{dy}{dx}) = 3x^2 + 3y^2\frac{dy}{dx}$

$3(-1+1)^2(1 + \frac{dy}{dx}) = 3(-1)^2 + 3(1)^2\frac{dy}{dx}$

$0 = 3 + 3\frac{dy}{dx}$

$-3 = 3\frac{dy}{dx}$

$-1 = \frac{dy}{dx}$

$$\underline{3y}\frac{dy}{dx} + \underline{2x^2}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(3y + 2x^2\right)$$

d) $3x^4 = (2xy - 1)^3$

$$\frac{12x^3}{3(2xy-1)^2} = \frac{3(2xy-1)^2 \cdot \left(2x\frac{dy}{dx} + 2y\right)}{3(2xy-1)^2}$$

$$\frac{12x^3}{3(2xy-1)^2} = 2x\frac{dy}{dx} + 2y$$

$$\frac{12x^3}{3(2xy-1)^2} - \frac{2y}{1} = 2x\frac{dy}{dx}$$

$$\frac{1}{2x} \cdot \frac{12x^3 - 2y\left(3(2xy-1)^2\right)}{3(2xy-1)^2} = 2x\frac{dy}{dx} \cdot \frac{1}{2x}$$

$$\frac{12x^3 - 2y\left(3(2xy-1)^2\right)}{2x \cdot 3(2xy-1)^2} = \frac{dy}{dx}$$

$y^4 = y^2 - x^2$

Alt

$\frac{dy}{dx} = \frac{-2x}{-2y+4y^3}$

$1-2y^2=0$

$1=2y^2$

$\frac{1}{2}=y^2$

$\pm\sqrt{\frac{1}{2}}=y$

③ $4y^3 \cdot \frac{dy}{dx} = 2y\frac{dy}{dx} - 2x$

$\frac{dy}{dx} = \frac{2x}{2y-4y^3}$

horiz tangent when $\frac{dy}{dx}=0$

$2x=0 \Rightarrow x=0$

when $x=0 \rightarrow y^4=y^2$

$y^4-y^2=0$

$y^2(y^2-1)=0$

$y=0$; $y=\pm 1$

horiz @ $(0,-1)$, $(0,0)$, $(0,1)$

Vert. tangent $\frac{dy}{dx}$ undef

$2y-4y^3=0$

$2y(1-2y^2)=0$

$y=0$ $\quad y=\pm\frac{1}{\sqrt{2}}$

$\downarrow$

$\cancel{(0,0)}$

when $y=\pm\frac{1}{\sqrt{2}} \rightarrow \frac{1}{4}=\frac{1}{2}-x^2$

$x^2=\frac{1}{4}$

$x=\pm\frac{1}{2}$

So vertical @ $(\pm\frac{1}{2}, \pm\frac{1}{\sqrt{2}})$ (4 points)

@ $(0,0)$ $4y^3$ approaches 0 much faster than $2x$ or $2y$ so

$\frac{dy}{dx} \rightarrow \frac{2x}{2y} = \pm\frac{2}{2} = \pm 1$ (b/c graph approaches $y^2 \approx x^2 \rightarrow y \approx \pm x$ near 0)

So graph is neither horiz. or vert. @ $(0,0)$

3) $y^4 = y^2 - x^2$

$$4y^3\frac{dy}{dx} = 2y\frac{dy}{dx} - 2x$$

$$-2y\frac{dy}{dx} \quad -2y\frac{dy}{dx}$$

$$4y^3\frac{dy}{dx} - 2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx}\left(4y^3 - 2y\right) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{4y^3 - 2y}$$

(4) a) $y = x^4 - 3x^2 + 4$

$y' = 4x^3 - 6x$

$y' = 0 \rightarrow 2x(2x^2 - 3) = 0$

$x = 0$; $x = \pm\sqrt{\frac{3}{2}}$ outside domain

$y(-1) = 2$ Abs. min

$y(0) = 4$ Abs. max

$y(1) = 2$

b) $y = x^3 + 6x^2 + 9x + 3$

$y' = 3x^2 + 12x + 9$

$y' = 0 \rightarrow 3(x^2 + 4x + 3) = 0$

$x = -1, -3$

$y(-4) = -64 + 96 - 36 + 3 = -1$

$y(-3) = -27 + 54 - 27 + 3 = 3$

$y(-1) = -1 + 6 - 9 + 3 = -1$

$y(0) = 3$

Abs min : -1, Abs Max 3

(5) a) $y = x^4 - 10x^2 + 9$

$y' = 4x^3 - 20x = 4x(x^2 - 5)$

$x = 0$ $x = \pm\sqrt{5}$

y': $-$ | $+$ | $-$ | $+$ at $-\sqrt{5}$, 0, $\sqrt{5}$

y increasing for $x \in (-\sqrt{5}, 0) \cup (\sqrt{5}, \infty)$

y decreasing for $x \in (-\infty, -\sqrt{5}) \cup (0, \sqrt{5})$

b) $y = \frac{-x}{x^2 + 4}$

$y' = \frac{(x^2+4)(-1) - (-x)(2x)}{(x^2+4)^2}$

$y' = \frac{-x^2 - 4 + 2x^2}{(x^2+4)^2} = \frac{x^2 - 4}{(x^2+4)^2}$

$y' = 0$ when $x = \pm 2$

y' undef $\emptyset$

y': $+$ | $-$ | $+$ at -2, 2

$\nearrow \quad \searrow \quad \nearrow$

y increasing on $x \in (-\infty, -2) \cup (2, \infty)$

y decreasing for $(-2, 2)$